## UNIT 2: POLYNOMIAL FUNCTIONS

## DAY 1: DEGREES OF POLYNOMIAL FUNCTIONS - SHAPING UP

## Investigating Shapes of Polynomial Functions Using Graphing Calculators

Polynomial Function: is a continuous function in one variable with whole number exponents in descending order of degree. The degree of the function is the highest exponent.

$$
\begin{aligned}
& y=9 \\
& y=5 x+9 \\
& y=3 x^{2}+5 x+9 \\
& y=x^{n}
\end{aligned}
$$

Has degree 0 - Horizontal Line
Has degree 1 - Linear Function
Has degree 2 - Quadratic Function (Parabola)

Has degree n

A function is a relation with the following features:

- For every $x$-value there is only one $y$-value
- The graph will pass the Vertical Line Test


## Part A: Shaping Up

Consider the following family of functions. Using the graphing calculators as needed with the suggested window: $-10 \leq x \leq 10,-10 \leq y \leq 10$. Note: ZOOM 6 will automatically set the window for you.

| Constant <br> (Degree 0) | Linear <br> (Degree 1) | Quadratic <br> (Degree 2) | Cubic <br> (Degree 3) | Quartic <br> (Degree 4) |
| :--- | :--- | :--- | :--- | :--- |
| $*$ | $*$ | $*$ | $*$ | $*$ |
|  |  |  |  |  |
|  |  |  |  |  |

1a) What do the equations of the functions in each family have in common?
b) What do all the graphs in each family have in common? Include sketches with your explanation.

2a) Predict the shape of the graph of a typical polynomial function of degree 5 .
b) Graph the function $y=x^{5}-5 x^{3}+4 x-3$ to check your prediction. Sketch the result.

## Part B: Who's Got the Power?

1a) Graph the following functions on a graphing calculator using a window of $-5 \leq x \leq 5$ and $-100 \leq y \leq 100$.

Domain: is the behaviour of $x$ values. $D=\{x \mid x \in r\}$ because it is continuous to infinity. Range: is the behaviour of $y$ values.

SET A:

$$
\begin{aligned}
& y=40 x+1 \\
& y=3 x^{3}-2 x^{2}+3 x-7 \\
& y=2 x^{5}-2 x^{2}-3
\end{aligned}
$$

SET B

$$
\begin{aligned}
& y=-30 x+2 \\
& y=-2 x^{3}+2 x^{2}+3 x-7 \\
& y=-x^{5}-2 x^{2}+3
\end{aligned}
$$

b) Complete these statements:

If a function has an odd degree and a leading coefficient that is positive, then the graph will...

If a function has an odd degree and a leading coefficient that is negative, then the graph will...

2a) Graph the following functions on a graphing calculator using a window of $-5 \leq x \leq 5$ and $-100 \leq y \leq 100$.

Domain: is the behaviour of $x$ values. $D=\{x \mid x \in r\}$ because it is continuous to infinity.
Range: is the behaviour of $y$ values.

SET A:

$$
\begin{aligned}
& y=2 x^{2}+1 \\
& y=3 x^{4}-2 x^{2}+3 x-7
\end{aligned}
$$

SET B

$$
\begin{aligned}
& y=-2 x^{2}+3 x-7 \\
& y=-x^{4}+2 x^{2}+3
\end{aligned}
$$

b) Complete these statements:

If a function has an even degree and a leading coefficient that is positive, then the graph will...

If a function has an even degree and a leading coefficient that is negative, then the graph will...
3. Is it fair to say that you can tell the general shape of the graph of a polynomial function just by looking at one term in the equation? Which term would you look at? Explain. This term is called the "dominant term".

HW: Handout "4.1 Polynomial Functions" and pg. 197 \# 2, 5

The following is a list of criteria in order for a given relation to be classified as a POLYNOMIAL FUNCTION.

1. the relation is expressed in terms of one variable (i.e. x)
2. each term in the relation is a power with the variable as the base
3. the degree of each term is a whole number
4. the relation is a function (i.e. Vertical Line Test)
5. the domain of the relation is any real number
6. the range of the relation is any real number OR it has an upper of lower boundary (not both)
7. the relation cannot have any restrictions

HW: Handout "4.1 Polynomial Functions" and pg. 197 \# 1
HANDOUT: On the reverse side of this page is a list of examples of polynomial functions and a list of examples of non-polynomial functions.

Identify using the criteria, why each of the non-examples are not polynomial functions.

|  | Odd-Degree Polynomial |  | Even-Degree Polynomial |  |
| :---: | :---: | :---: | :---: | :---: |
| Leading Coefficient | Positive | Negative | Positive | Negative |
| Sketch |  |  |  |  |
| Domain | $\{x \in R\}$ |  | $\{x \in R\}$ |  |
| Range | $\{y \in R\}$ |  | $\{y \in R, y \geq a\}$ | $\{y \in R, y \leq a\}$ |
| Maximum/ Minimum Value | Neither a maximum value nor a minimum value |  | Minimum value is a Maximum value is a |  |

Domain:

Range:
(Relative) Maximum Point:
(Relative) Minimum Point:

Symmetry:

## Examples:

## 1. Graph the function defined by

Determine the coordinates of any relative maximum or relative minimum points. What is the domain and range of the function?

2. Graph the function defined by Determine the coordinates of any maximum or minimum points. What is the domain and range of the function?

## Relative max and min:



HW: Text pg. 203 \# 1-5

Unit 2: Assignment 1 - Match the Graphs

## DAY 5: TEST \#1

Unit 2: Test \#1

## DAY 6: WHAT ROLE DO FACTORS PLAY?

HANDOUT: What Role Do Factors Play?

HW: Handout - Factoring in our Graphs

## DAY 7: RELATING POLYNOMIAL FUNCTIONS

The $x$-intercepts of the graph of a polynomial function $y=f(x)$ are the zeros of the function and the roots of the corresponding equation $f(x)=0$.

Polynomial functions that have the same zeros...

## Examples:

1. Consider the family of quadratic equations, where each function has zeros at -2 and 2 .
a) White the equation of the family, and then state two functions that belong to the family.
b) Determine the equation of the member of the family that passes through $(1,4)$ and graph.

2. Sketch the graph of each function.
a)

b)


HW: HANDOUT - Polynomial Functions in Factored Form - Summary

- Finding the Equation of Polynomial Functions (not the extra text work)


## DAY 8: RECIPROCAL FUNCTIONS

The reciprocal of the number $x$ is the number , provided that $x \neq 0$.
Similarly the reciprocal of a function $f(x)$, is the function , provided that $f(x) \neq 0$.

Examples:

1. For the function $y=x-2$, the reciprocal function is , where $x \neq 2$.
a) Graph these functions.
b) Describe the symmetry of the reciprocal function.


HW: Unit 2: Assignment 2 - Reciprocal Functions or Unit 2: Review Assignment

DAY 9: WORK PERIOD

Study for Unit Test.

DAY 10: UNIT TEST

